Discovery Activities for Basic Algebra II

Understanding Algebra through Problem Solving

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Introduction

This supplement contains 15 activities that will help you understand the core concepts of Algebra through problem solving. Each activity presents a problem situation and then guides you through the problem solving process to gain experience in applying the skills and ideas presented in Basic Algebra II. These guided discovery activities provide a hands-on learning experience in applying the main concepts of algebra to the world around us. You will learn to use mathematics as a tool to solve real life problems and gain a deeper appreciation of how to look at the world through mathematical eyes.

There are many ways to incorporate these activities into your instruction. At the beginning of class, the author likes to provide a brief (10-15 minutes) introduction to the mathematical concept or skill that is to be covered. The students then work collaboratively in small groups on these activities as active learning assignments. The instructor facilitates this collaborative learner experience by providing help to groups that get stuck while working through the hands-on activities. These activities can be used both in and out of class. The instructor can go through some of the activities with the whole class and then assign others to finish outside class as small projects.
Discovery Activity 1: The Cost of College Tuition
Marissa is going back to school to get her teaching certificate. She needs 12 additional credits to obtain certification and plans to attend one of the two local colleges. City College costs $70 per credit plus a $150 one-time registration fee, while State College costs $95 per credit and has no registration fee. Assuming both colleges have a similar academic reputation, which is the better choice based on cost? Go to part a to begin.

a. Show the computational steps necessary to find the cost of taking one 3-credit course at City College and repeat the process for one 4-credit course at the same school.

b. Suppose the total number of credits a student takes is represented by the variable n and the cost to attend City College is represented by the variable C. Write a formula that expresses the cost C in terms of the number of credits taken n.

c. Show the computational steps necessary to find the cost of taking one 3-credit course at State College and repeat the process for one 4-credit course at the same school.

d. If the cost to attend State College is represent by S, then write a formula that expresses the cost S in terms of the number of credits taken n.

e. Complete the table below that gives the total cost a student pays at each college, including registration fee, for taking any number from 3 to 12 credits during a semester. Verify your results by creating a table in your graphing calculator.

<table>
<thead>
<tr>
<th>Number of Credits $n$</th>
<th>Cost at City College $C$ (in $)</th>
<th>Cost at State College $S$ (in $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
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<tr>
<td>7</td>
<td></td>
<td></td>
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<tr>
<td>8</td>
<td></td>
<td></td>
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<tr>
<td>9</td>
<td></td>
<td></td>
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<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If it cost Marissa’s friend Megan $1130 to attend City College last semester, then how many credits did she take? Set up and solve the linear equation that will give the answer to this question. Use your graphing calculator to verify your solution by extending the table you constructed in part e.

Based on the data in the table, when does City College cost the same as State College?

Suppose both colleges allow a student to take up to 18 credits. Use inequality notation or interval notation to answer the following questions.

- Over what range of credits does City College cost more than State College?
- Over what range of credits does City College cost less than State College?

Symbolically the cost at City College should be equal to the cost at State College when \( C = S \). Verify your answer to part g, by setting the right side expressions, from the equations in parts b and d, equal to each other and then solve for \( n \).
Discovery Activity 2: Signing the Best Contract
Suppose that Megan wrote a romance novel during her last year in school, and two publishers extend her offers to sign the manuscript for publication. Company A offers her a $10,000 sign-on bonus and 15% of the book’s total sales. Company B offers her a $5,000 sign-on bonus and 18% of the book’s total sales. Assuming both companies would spend about the same amount on advertising and promoting the book, which publisher should she sign with? Go to part a.

a. Since we do not know the total sales revenue in dollars, complete the following table to see how Megan’s income in dollars depends on total sales of the book in dollars.

<table>
<thead>
<tr>
<th>Total Sales ($)</th>
<th>Megan’s Income Company A ($)</th>
<th>Megan’s Income Company B ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>125,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Use the table above to compare how Megan’s income will change, depending on which company she uses, as total sales increase. Describe any patterns you observe.

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Let $x$ be the total sales (in $) of the book and $y$ be the income (in $) that Megan would receive. Using these variables, setup two equations for the incomes from Companies A and B. *Hint: Megan’s income $y$ is the fixed bonus plus the percent of total sales $x$.*

Company A: 

Company B: 

d. Verify your results to part a by creating a table in your graphing calculator using the two equations for companies A and B.
e. Based on the data in the table, estimate the total sales that will produce the same income from each company? You are not trying to find an exact answer, however your approximation will be more accurate if you change the graphing calculator table settings to increments (steps) smaller than 25,000. Explain the process you used to arrive at your answer.

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f. To find the exact amount of total sales that will produce the same income from each company, set the right side of the equation of Company A equal to the right side of the equation for Company B, then solve this new equation for \( x \). Round to the hundredths place.

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g. Based on all of the information, how would you advise Megan on selecting a publisher? Be sure to support your advice with quantitative reasoning.
Discovery Activity 3: Maximizing Space for Bruin

You have 200 feet of fencing available and need to build a rectangular pen. Your goal is to find the dimensions that would give the family dog, Bruin, the maximum area to live in. Suppose you decide to build a rectangular pen along the back of the house so that you only need to fence in three sides (i.e., the house is the fourth side and would not need fencing).

a. Draw a sketch of the rectangular pen in its location against the back of the house. Label the side opposite the house as $y$ and the two sides connecting the $y$-side to the house as $x$.

b. Write an equation for the fencing needed using only variables $x$ and $y$ and the 200 feet of fencing that will enclose the three sides of the pen.

c. The equation in part b involves two related variables $x$ and $y$. Isolate $y$ in the equation from part b. Your equation for $y$ should be written in terms of $x$.

d. The area of a rectangle is length times width, and so we have the formula $A = x \cdot y$ to figure out how to give Bruin the maximum play space. Substitute the expression you obtained for $y$ from part c into the given area formula so that $A$ is written only in terms of the variable $x$. 

e. We wish to construct a table that gives the width, length and area of different rectangular pens. Input the two equations from parts c and d into your graphing calculator to construct the following table and observe how different dimensions change the value of the area of the pen. Be sure to have the calculator’s table set to start at 10 and increase in increments of 10.

<table>
<thead>
<tr>
<th>Width, ( x ) (feet)</th>
<th>Length, ( y ) (feet)</th>
<th>Area, ( A ) (sq. feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f. Based on the table above, find the maximum area for Bruin and state the dimensions that will produce the maximum area. Be sure to include units with your answers.

g. Describe any patterns you observe in the table above. Indicate how the length and area change as the width increases from 10 feet to 90 feet.
**Discovery Activity 4: Slow Down and Save Money**

The speed at which you drive a car on the highway can affect the car’s fuel economy. For example, the July 2008 Consumer Reports magazine contained the following data from tests they performed on the Toyota Camry. The speed is given in miles per hour (mph) and the fuel economy in miles per gallon (mpg).

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>55</th>
<th>65</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Economy (mpg)</td>
<td>40</td>
<td>35</td>
<td>30</td>
</tr>
</tbody>
</table>

**a.** What are the two related variables in the above situation?

**b.** The value of one of these variables depends on the value of the other. Explain which variable depends on the other. This is called the **dependent variable**.

**c.** The **independent variable** determines the value of the dependent variable. What is the independent variable?

**d.** What are the units for the independent and dependent variables?

**e.** As the speed of the Camry increases by 10 mph, what happens to the fuel economy?

**f.** Assume that the Camry has a constant decrease in fuel economy at speeds between 50 and 90 mph inclusive. What is the constant rate of change in fuel economy with respect to speed? *Hint: The rate of change is the slope.*

**g.** What does the rate of change or slope indicate about the speed and fuel economy of the Camry?

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**h.** Letting $x$ represent speed and $y$ fuel efficiency, find a linear equation $y = mx + b$ that models this situation at speeds between 50 and 80 mph inclusive. *Hint:* Find $b$ by substituting the slope $m$, found in part $f$, and a given point $(x, y)$ from the data table into $y = mx + b$ and then solve for $b$.

**i.** Place your equation from part $h$ in a graphing calculator and use the table feature to find a table that starts at 50 and increases in increments of 5. Record the table below.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Economy (mpg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**j.** Use your graphing calculator to find the highway mileage of a Camry traveling at an average speed of 77 mph. Record the mileage below with units. There are many ways to obtain the answer; one way is to use the calculator’s table feature again.

**k.** Verify your answer to part $j$ algebraically by substituting the given input value of 77 into the equation found in part $h$ and evaluating the resulting expression.

**l.** Suppose you drove your new 2008 Camry on a 160 mile road trip and used 5 gallons of gas. Calculate the fuel efficiency or mileage for this trip. Include units in your answer.

**m.** Suppose during the trip in part $l$ you traveled at a steady speed but never looked at the speedometer, what was your average speed for the trip? First find the answer algebraically using the linear equation from part $h$. Then verify your answer numerically using your graphing calculator table feature.
\textbf{n.} Graph this function over the interval $[50, 80]$ by plotting the points in the table from part \textit{i}. Be sure to include labels and scales for each axis and a title at the top.

\textbf{o.} Verify your work in part \textit{n} by graphing the equation from part \textit{h} using your graphing calculator. Be sure to change the calculator’s window setting before graphing.
Discovery Activity 5: Speed Trap
Suppose you live in an area where the fine for a speeding violation is $50 plus $10 for every mile per hour over the posted speed limit. The maximum fine that can be given is $500.

a. A police officer uses a radar gun to catch a driver speeding at 35 mph over the speed limit. What fine can the driver receive for speeding?

b. If the speed limit is 65, what was the speed of the driver in part a?

c. You are driving on a highway with a posted speed limit of 65 mph. You are going 70 mph but most of the surrounding cars are moving faster. Complete the table below to see what types of fines could be given for speeding.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. What are the two related variables in this problem situation? Also state the units for each variable.

e. State which variable is the independent variable or input and which is the dependent variable or output. Explain your reasoning using complete sentences.

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f. If \( x \) represents the speed of the car in mph, then find a variable expression, in terms of \( x \), that represents the speed amount over the limit of 65 mph.

g. Let \( y \) represent the fine amount in dollars. Write a linear equation that gives the fine \( y \) in terms of the speed \( x \). Simplify your equation so it is written in the form \( y = mx + b \).

\textit{Hint}: Use the expression developed in part \( f \).
h. State the rate of change or slope of the equation in part g and then interpret its meaning in terms of the problem situation. *Hint:* think about what the change in $x$ and $y$ represent.

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p. State the $y$-intercept of the equation in part $g$ and explain why it has no practical meaning in this problem situation.

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q. Place your equation from part $g$ in your graphing calculator and use the table feature to find a table that starts at 60 and increases in increments of 1. What is the first $x$-value or speed for which the equation makes sense? Explain.

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i. What is the **practical domain** for the equation you created in part $g$? *Hint:* practical domain consists of all input values ($x$) that make sense in terms of the problem situation. Recall that $500$ is the maximum fine that can be given out for speeding.

j. What is the **practical range** for the equation you created in part $g$? *Hint:* practical range consists of all output values ($y$) that correspond to the input values found in part $i$. 
Suppose a police officer who pulled you over for speeding gave you a $180 ticket but never indicated how fast your car was traveling. If the officer was using a radar gun, what speed did he use in writing your ticket amount? Assume the speed limit is 65 mph. First find the answer by solving an equation algebraically. Then verify your answer using the table feature of your graphing calculator. Show the algebraic steps below.

l. Graph the speeding-fine equation from part g over the practical domain. Be sure to include labels and a scale on each axis.

m. Verify your work in part l by graphing the equation from part g using your graphing calculator. Be sure to change the calculator’s window setting before graphing.
**Discovery Activity 6: The Bookstore Markup**

Suppose the college bookstore obtains books at a wholesale cost and then mark up the wholesale cost by 35% to obtain the retail price (the price the consumer pays before taxes.)

**a.** What are the two related variables in the above situation? *Hint:* The variables are unknown quantities that can vary or change.

**b.** The value of one of these variables depends on the value of the other. Explain which variable depends on the other. (This is called the dependent variable or output variable.)

**c.** The dependent variable’s value is determined by the value of the independent variable or input variable. What is the independent variable?

**d.** A functional relationship exists between two variables if for each value of the independent variable there is exactly one corresponding value of the dependent variable. In other words, each input value has just one output value. Does a functional relationship exist in this problem situation? Explain.

**e.** The markup is the amount that the bookstore increases the wholesale cost by to obtain the retail price. If we let the wholesale cost be $x$, then how can we define the markup in terms of $x$?

**f.** If we let the retail price be $y$, then express the relationship between $x$ and $y$ with an equation that gives $y$ in terms of $x$. 
g. If \( y \) is a function of \( x \), we can use function notation and say \( y = f(x) \), which is read, “\( y \) equals \( f \) of \( x \)” and means that \( y \) depends on \( x \). Write the function from part \( f \) again using the function notation, \( f(x) \). Hint: write the same equation as part \( f \) but use \( f(x) \) instead of \( y \).

h. Use the retail function found in part \( g \) to find \( f(70) \).

i. Explain what the answer to part \( h \) means in terms of the bookstore situation.

j. Find the \( x \)-value that satisfies the equation, \( f(x) = 67.5 \). Hint: Set the function rule found in part \( g \) equal to 67.5, then solve the resulting equation for \( x \).

k. Explain what the answer to part \( j \) indicates about the bookstore situation.

l. What is the rate of change or slope? Explain what this value means in terms of the bookstore situation.
**m.** Enter the function from part \( g \) into a graphing calculator and build a table that shows the retail prices for wholesale costs from $10 to $80, increasing in increments of $10.

<table>
<thead>
<tr>
<th>( x ), Wholesale ($)</th>
<th>( y ), Retail ($)</th>
</tr>
</thead>
</table>

**n.** Graph the ordered pairs from the table in part \( m \) as points on the rectangular coordinate system below and then connect the points with a straight line to display the linear function graphically. Be sure to include a title and labels and scale for each axis. Check your work on a graphing calculator using the graph feature and an appropriate window.

**o.** Assuming a 5% sales tax, construct a second function, named \( g \), that will given the price a consumer pays after taxes in terms of the wholesale cost \( x \). Put the function in the simplest form possible. \( \text{Hint: Price including tax} = \text{Retail Price} + \text{Tax} \).

\[
g(x) =
\]

**p.** Find \( g(89) \).

**q.** Explain what the answer to part \( p \) means in terms of the bookstore situation.
Discovery Activity 7: Cricket Measures Temperature

Sitting out on a warm summer day you hear the sound of crickets. As the day progresses you feel the temperature getting warmer and notice that the crickets are chirping at a faster rate. You access a website of The National Weather Service that allows the user to enter the number of cricket chirps in 15 seconds and then the site will display the temperature in degrees Fahrenheit (°F) and degrees Celsius (°C). So you begin to construct a table and record the following data.

<table>
<thead>
<tr>
<th>Chirps per 15 seconds</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>55</td>
<td>60</td>
<td>65</td>
<td>70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Describe the two related variables.

b. If the number of chirps is used to determine the temperature, then state the independent variable or input and the dependent variable or output.

c. Observe the data table above and describe any patterns you see. Also, state what basic type of function is displayed by this data.

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d. If the pattern in the table above continues, find the next 5 ordered pairs. Record these values in the table above.

e. If crickets are usually silent when the temperature falls below 55, then what is a possible (practical) domain for the cricket relation above? Assume that the temperature rarely goes above 95° F in the environment where the cricket lives.

f. Based on the domain from part e, what is the corresponding (practical) range?
g. Is the temperature a function of the number of chirps? Explain.

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h. Find the rate of change or slope of this data set.

i. What does the rate of change or slope indicate about the problem situation?

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j. If \( c \) represents the number of chirps per 15 seconds and \( T \) represents the temperature in degrees Fahrenheit, then find an equation or formula that gives the temperature, \( T \) in terms of the number of chirps, \( c \). Use function notation letting \( T = T(c) \).

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k. Find \( T(32) \) and explain what the result indicates about the problem situation.

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l. Verify that the formula developed in part \( j \) is correct by entering it into a graphing calculator and constructing a table that starts at 15 and increases in increments of 5. Compare the calculator’s table to the given table on the previous page.
m. Graph the data in the given table as points on the rectangular coordinate system below and then connect the points with a line. Be sure to include labels and scales along each axis and an appropriate title at the top. Check your work on a graphing calculator.

n. The air temperature can be converted from Fahrenheit to Celsius using the following formula.

\[ T_c = \frac{5}{9}(T_f - 32) \]

where \( T_c \) is temperature in degrees Celsius and \( T_f \) is temperature in degrees Fahrenheit

Copy the table you constructed previously and then enter the above formula into a graphing calculator to find the temperatures in degrees Celsius. Set your table so that the input values are the temperatures in degrees Fahrenheit and record the output temperatures in degrees Celsius in the third row, rounding to the tenths place.

<table>
<thead>
<tr>
<th>Chirps per 15 seconds</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>55</td>
<td>60</td>
<td>65</td>
<td>70</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

o. Use a graphing calculator to graph the ordered pairs in the first and third rows to display the Celsius temperature as a function of chirps per 15 seconds.

p. According to the Old Farmer’s Almanac certain crickets can estimate the temperature in degrees Fahrenheit if we count the number of chirps in 14 seconds and then add 40. Letting \( c \) represent the number of chirps per 14 seconds and \( T_f \) the temperature in degrees Fahrenheit, find a formula to model the Almanac’s rule.
q. Compare the Old Farmer’s Almanac rule in part p to the National Weather Service rule in part j. How do the two rules compare in estimating temperature by counting the number of chirps made by a cricket?

r. Also, according to the Old Farmer’s Almanac certain crickets can estimate the temperature in degrees Celsius if we count the number of chirps in 25 seconds, divide by 3, and then add 4. Let \( c \) represent the number of chirps per 25 seconds and \( T_e \) the temperature in degrees Celsius, find a formula to model the Almanac’s rule.
Discovery Activity 8: The High Cost of Driving
The cost of fuel for transportation is a big ticket item for most consumers. Since you probably need to fill up your vehicle with gas more often than you like, it is good idea to look around for the best price per gallon.

a. There are two input variables that determine the cost (output) to fill a vehicle’s tank with fuel. What are they? Be specific using complete sentences.

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b. If we assume that the price of gas per gallon is $2.75, then one of the input variables in part a will become a constant. The value of this constant will not vary throughout the problem. The cost to fill a vehicle’s tank with fuel is now dependent on only one variable, the number of gallons of gas pumped. Complete the table below to see the relationship between the two variables.

<table>
<thead>
<tr>
<th>Number of Gallons</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Fuel ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Is the cost of fuel a function of the number of gallons pumped? Explain.

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d. Write a verbal statement (words) that describes how the cost of fuel is determined.

e. Let g be the number of gallons of gasoline pumped and c represent the cost of the fuel. Translate the verbal statement in part d into a symbolic statement (an equation) that expresses c in terms of g.
\( f. \) Write the equation from part \( e \) using function notation by letting \( c = f(g) \). Then evaluate \( f(8) \) and write the result as an ordered pair.

\( g. \) Next evaluate \( f(12) \), express the result as an ordered pair, and then write a sentence describing its meaning.

\( h. \) Can any number be substituted for the input variable \( g \) in the cost of fuel function? Describe the values of \( g \) that make sense, and explain why they do.

\( i. \) The values of \( g \) that make sense in terms of the problem situation are the practical domain. Assuming 20 gallons is the maximum capacity of your gas tank; express the practical domain using set-builder notation or interval notation.

\( j. \) Enter the function from part \( e \) into a graphing calculator and build a table that shows the cost of fuel for number of gallons purchased from 2 to 20, increasing in increments of 2. Round the cost of fuel to the nearest penny.

<table>
<thead>
<tr>
<th># of Gallons, ( g )</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Fuel, ( c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( k. \) Based on the domain from part \( i \), what is the corresponding (practical) range?
l. Is the cost of fuel a function of the number of gallons purchased? Explain.

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m. Graph the cost of fuel function over its practical domain. Include scales, labels, and title.

[Graph of cost of fuel function]

n. What is the rate of change or slope of the function expressed verbally in part d, symbolically in part e, numerically in part j, and graphically in part m.

o. What does the rate of change or slope indicate about the problem situation?

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p. What are the domain and range of a generic function \( f(x) = 2.75x \) that has no connection to the context of cost of fuel and number of gallons purchased?
**Discovery Activity 9: Music Buying Trends**

According to Recording Industry Association of America digital music sales have been growing, with digital downloads increasing 30% from 2007 to 2008. Simultaneously sales in physical music items, such as CDs, cassettes, and LPs, have been declining. The table below gives number of CDs shipped per year from 2000 to 2008.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDs shipped (millions)</td>
<td>942.5</td>
<td>881.9</td>
<td>803.3</td>
<td>746.0</td>
<td>767.0</td>
<td>705.4</td>
<td>619.7</td>
<td>511.1</td>
<td>384.7</td>
</tr>
</tbody>
</table>

**Change (millions)**

- **a.** Let time be the number of years since 2000, so time 0 is 2000, 1 is 2001, etc. Enter time (in years since 2000) and number of CDs shipped (in millions) into a graphing calculator and find the change in CDs shipped per year. Record the change in the third column.

- **b.** Describe what the third column indicates about CD shipments from 2000 to 2008.

- **c.** In the graph below, plot the data pairs in the first two rows of the given table. Label each axis and place an appropriate title at the top.
**d.** Observe the graph in part c, then describe any trend you see in the number of CDs shipped during the years 2000 to 2008.

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**e.** Although the data points due not lie in a straight line, there is a linear trend to the data. Use a ruler or straightedge to draw a line of best fit. This is a line that comes as close as possible to all the data points but does not necessarily go through any given data points. Try to draw your line so that roughly half the data lies above the line and half lies below.

**f.** Use the grid on the graph to help select two points from your best fit line you constructed in part e. It is best if the two points are not too close together. In the space below, record the ordered pairs that correspond to the points you selected.

**g.** Use the points selected in part f to find the slope of your best fit line.

**h.** Find the y-intercept of your best fit line.

**i.** Find the equation of your best fit line.

**j.** Use your calculator’s linear regression command to find the least squares regression line for the data. Record the equation for the regression line below, rounding the slope and y-intercept to the tenths place. The linear equation should be in the form $y = mx + b$ or $f(x) = mx + b$. 
**k.** Use a graphing calculator to draw a scatterplot of the given data pairs: years since 2000 and CDs shipped (in millions). Then graph the least squares regression line found in part \( j \). Record your observations on how well the line approximates the data points visually.

**l.** Associated with each regression line is a number between \(-1\) and \(1\) inclusive called the correlation coefficient. Whenever you use the calculator to find the regression line, the calculator computes the correlation coefficient and stores it as variable \( r \). Find this value and record it below, rounding to the thousandths place.

Values of \( r \) that are close to \(1\) indicate a strong positive correlation between input and output. This means the quantities most likely exhibit a linear relationship and that the regression line has a positive slope.

Values of \( r \) that are close to \(-1\) indicate a strong negative correlation between input and output. This means the quantities most likely exhibit a linear relationship and that the regression line has a negative slope.

Values of \( r \) near zero indicate little correlation and suggest that there is no linear relationship between input and output. For a sample of size 9, there is no correlation if \( r \) is between \(-0.666\) and \(0.666\).

**m.** Based on the information above, what does the value of \( r \) found in part \( l \) tell you about the relationship between year and number of CDs shipped?

**n.** Use the least squares regression line to predict the number of CDs shipped in 2009.

**o.** Use the least squares regression line to predict the year that the number of CD’s shipped goes below 200 million?
Discovery Activity 10: U.S. Population Growth

We can estimate future population growth by studying past trends over time. The table below contains data from the U.S. Census Bureau on annual estimates of the resident population of the United States in millions from July 1, 2000 to July 1, 2008.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (millions)</td>
<td>282.2</td>
<td>285.0</td>
<td>287.7</td>
<td>290.2</td>
<td>292.9</td>
<td>295.6</td>
<td>298.4</td>
<td>301.3</td>
<td>304.1</td>
</tr>
<tr>
<td>Change (millions)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**a.** Let time be the number of years since 2000, so that time 0 is 2000, time 1 is 2001, etc. Enter time (in years since 2000) and population (in millions) into a graphing calculator and find the change in population each year from 2000 to 2008. Record the change in the third column of the table above.

**b.** Describe the change in the population from 2000 to 2008 based on the results from part **a**.

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**c.** In the graph below, plot the data points you entered into your graphing calculator but do not connect the points. Label each axis and place an appropriate title at the top.
d. Describe the change in the U.S. population from 2000 to 2008 based on the graph. Also, explain why it is helpful to have the vertical axis begin at 280 instead of 0.

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e. Use your graphing calculator to find a linear regression equation that models the data you plotted. Round the slope and y-intercept to the hundredths place. Also, record the correlation coefficient \( r \), rounding to four decimal places.

f. Use a graphing calculator to draw a scatterplot of the given data pairs: years since 2000 and U.S. population (in millions). Then graph the least squares regression line found in part e. Based on visual inspection of the graph and the correlation coefficient \( r \), describe how good a fit the regression equation is to the given data.

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g. Explain what the two variables represent in your regression equation. Indicate the independent and dependent variables.

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\textbf{h.} Explain what the slope and \textit{y}-intercept mean in terms of population and time.

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\textbf{i.} Letting \(t\) represent time in years since 2000 and \(P(t)\) represent the corresponding population size, express the regression equation from part \(e\) using function notation. Then find \(P(10)\), rounding your answer to the tenths place.

\textbf{j.} Explain what \(P(10)\) means in terms of population and time.

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\textbf{k.} In what year does the model predict the population will reach 320 million?

\textbf{l.} Explain why it is important to analyze the population data of a country, state or town.

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Discovery Activity 11: Keeping Bruin Out
Suppose you want to plant a rectangular-shaped vegetable garden, but you need to protect it from your neighbor’s crazy dog, Bruin. Although Bruin is kept in a pen, his owners often forget to latch the gate securely. You have not seen the dog escape yet but plan on enclosing the perimeter of your garden with 18 meters of wire fencing, just in case. Your goal is to build a garden with dimensions that will produce the maximum amount of area.

a. Draw a picture of your garden labeling the length \(x\) and the width \(y\).

b. To develop an equation, use the formula for perimeter of a rectangle given below. Substitute the 18 meters of fencing for the Perimeter \(P\) and then isolate \(y\) on one side of the equation.

\[
Perimeter = 2 \cdot length + 2 \cdot width \\
P = 2x + 2y
\]

c. Develop a second equation by substituting the expression for \(y\), found in part \(b\), in the area equation given below.

\[
Area = length \cdot width \\
A = x \cdot y
\]

d. To better understand the rectangular dimensions that will give maximum area, complete the following table with the aid of a graphing calculator. \textit{Hint}: Enter the equations for width and area, found in parts \(b\) and \(c\), into your calculator, and then build the table.

<table>
<thead>
<tr>
<th>Length, (x) (meters)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width, (y) (meters)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area, (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
e. Describe any patterns you observe in the table from part d.

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f. If the length and width must be counting numbers, then what dimensions produce a maximum area for the garden and what is that area. Explain.

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g. Use the data in the table in part d to graph the area equation from part c. Be sure to label each axis and give the graph a title.
h. Verify the dimensions (length, $x$ and width, $y$) that are needed to build a rectangular garden with an area of 20 square meters, by substituting 20 for $A$ in the equation obtained in part c. Solve this quadratic equation by factoring. Hint: First put the equation in standard form, i.e., $ax^2 + bx + c = 0$.

i. Can you build a garden with an area greater than 20 square meters? Use the table feature of a graphing calculator to complete the following table.

<table>
<thead>
<tr>
<th>Length, $x$ (meters)</th>
<th>4.1</th>
<th>4.2</th>
<th>4.3</th>
<th>4.4</th>
<th>4.5</th>
<th>4.6</th>
<th>4.7</th>
<th>4.8</th>
<th>4.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width, $y$ (meters)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area, $A$ (sq meters)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

j. If the length and width can be any real numbers, then what dimensions produce a maximum area for the garden and what is that area. Explain.

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h. Explain why the area of a rectangular garden is a function of the length. Also, what type of function is this?

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i. What is the practical domain of the area function?
What is the practical range of the area function?
Discovery Activity 12: Speed and Stopping Distance

Suppose while driving you see the brake lights of the car in front of you come on. If you assume the car ahead is going to stop and your brakes must be applied, then how far will your car travel before it comes to a stop? In other words, what is the stopping distance from the moment your brain receives the signal to brake until the car is no longer moving?

The table below contains data on this situation. Note that the total stopping distance is the sum of the reaction distance (distance traveled from the time you realize that you must brake until the brakes are applied) and the braking distance (distance traveled after the brakes are applied).

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Reaction Distance (feet)</th>
<th>Braking Distance (feet)</th>
<th>Total Stopping Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11</td>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>20</td>
<td>22</td>
<td>25</td>
<td>47</td>
</tr>
<tr>
<td>30</td>
<td>33</td>
<td>55</td>
<td>88</td>
</tr>
<tr>
<td>40</td>
<td>44</td>
<td>105</td>
<td>149</td>
</tr>
<tr>
<td>50</td>
<td>55</td>
<td>188</td>
<td>243</td>
</tr>
<tr>
<td>60</td>
<td>66</td>
<td>300</td>
<td>366</td>
</tr>
<tr>
<td>70</td>
<td>77</td>
<td>455</td>
<td>532</td>
</tr>
</tbody>
</table>

Stopping distances are based on tests made by U.S. Bureau of Public Roads. Driver reaction distance is based on the reaction time of \( \frac{3}{4} \) second.

a. Observe the pattern of the quantities in first two columns of the table above. What type of function exhibits this pattern? Explain.

b. Find an equation that gives the reaction distance \( R \) (in feet) as a function of the speed \( x \) (in miles per hour). Hint: Find the slope, \( m \), and \( y \)-intercept, \( b \), and then put in form \( R = mx + b \).
c. Enter the data from table columns 1 and 3 into your graphing calculator and plot the data with speed as the input or independent variable and braking distance as the output or dependent variable. Observe the pattern of the data points and then explain what type of function might be a good model for the data?

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d. Use the (quadratic) regression feature of your graphing calculator to find a symbolic model or equation giving braking distance \( B \) as a function of the speed \( x \). Round \( a, b, \) and \( c \) to the thousandths place.

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e. Since total stopping distance is the sum of reaction distance and braking distance, use the equations in parts \( b \) and \( d \) to find an equation that gives total stopping distance \( T \) as a function of speed \( x \).

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f. Create a table for the braking distance function \( B \) and the total stopping distance function \( T \). Round to the nearest foot.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Braking Distance (feet)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Stopping Distance (feet)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
g. Use the table in part f to construct a graph for the total stopping distance function $T$. Label and scale each axis and place an appropriate title at the top, center of the graph.

![Graph](image)

h. Observe the graph in part g and then explain the relationship between speed and stopping distance as speed increases.

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i. What is the total stopping distance for a car traveling at 75 mph? Round to the nearest foot.

j. Based on skid measurements at an accident scene, a car’s total stopping distance is 160 feet. Use your graphing calculator to estimate the speed of the car to the nearest mph.
Discovery Activity 13: The Shortest Distance Between Two Points
Currently, the only way to get from Center City to Pythagorean Airport is to travel 10 miles along Euclid Expressway, make a 90° turn onto Fermat Freeway, and go another 15 miles before reaching the airport. Because of traffic, the average speed along Euclid Expressway is 45 mph, and the average speed along Fermat Freeway is 40 mph. This produces a travel time that discourages many businesses from locating in Center City. A new route called Hypotenuse Highway has been proposed to connect Center City directly to the airport in a straight line. As part of the construction project team, you have been asked to complete the following tasks.

a. Draw a diagram (with labels) which illustrates the situation described above.

b. Find the distance from Center City to Pythagorean Airport via the proposed Hypotenuse Highway route. Round your answer to the nearest whole mile.

c. If an average speed of 60 mph can be expected when traveling along Hypotenuse Highway, what is the average time of travel between Center City and Pythagorean Airport? State your answer to the nearest tenth of an hour. Then convert the time to the nearest minute.
d. Find the time it currently takes to travel along the existing right angle route. Round your answer to the nearest tenth of an hour. Then convert the time to the nearest minute.

e. Approximately how many minutes will the new route save the average traveler?

f. What are some other factors to consider before going ahead with the project?

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g. The projected speed limit along Hypotenuse Highway is 60 mph, but vehicles will travel at different speeds depending on traffic and driving habits. Suppose we consider speed to be a variable represented by \( R \) (for rate in mph) and let \( T_h \) be time in hours. Find an equation that gives time \( T_h \) as a function of rate \( R \) when traveling the distance between Center City and Pythagorean Airport.

h. What constant can be multiplied by the equation in part g to obtain the time in minutes?

i. Let \( T_m \) represent the time in minutes. Find a second equation that gives the time in minutes \( T_m \) as a function of the rate \( R \) to travel the distance between Center City and Pythagorean Airport.
j. Enter the equations from parts g and i into a graphing calculator, and then use the table feature to complete the following table. This should give you an idea of the time of travel between Center City and Pythagorean Airport at different speeds. Round the time in hours to the nearest hundredth of an hour and the time in minutes to the nearest tenth of a minute.

<table>
<thead>
<tr>
<th>Rate (mph)</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hr)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time (min)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

k. Use the data pairs in the first and third rows of the table above to graph the time in minutes as a function of the rate in mph. Label each axis and place a title at the top. Verify your work by creating the same graph on your calculator.

l. Use the table and graph to describe how travel time changes as the speed or rate increases from 30 to 90 mph.

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Discovery Activity 14: The Pythagorean Short Cut

High Tech Place began with a few small start-up companies but has grown to become a large technology park with a dozen different companies. The only way to access the park from the main highway is to drive 5 miles along a straight road called Horizontal Way, and then make a 90° turn onto Vertical Line Road and drive a straight 3 miles until the road ends at High Tech Place. A new Access Road has been proposed to relieve heavy traffic and to shorten the commute. The Access Road would connect the highway to High Tech Place in a straight line.

a. Draw a diagram (with labels) which illustrates the situation described above.

b. Find the length to the nearest mile of the proposed access road. Show all work.

c. If an average speed of 50 mph can be expected when traveling along the new access road, what will be the average time of travel between the highway and High Tech Place? State your answer to the nearest hundredth of an hour. Then convert the time to the nearest minute.
d. The speed limit along Vertical Line Road and Horizontal Way is 35 mph. Find the time it currently takes to travel along the existing right angle route. Round your answer to the nearest hundredth of an hour. Then convert the time to the nearest minute.

e. Approximately how many minutes will the new road save the average traveler?

f. What must the project development team consider before moving ahead with the construction of a new access road?

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g. The projected speed limit along the access road is 50 mph, but vehicles will travel at different speeds depending on traffic and driving habits. Suppose we consider speed to be a variable represented by $R$ (for rate in mph) and let $T_h$ be time in hours. Find an equation that gives time $T_h$ as a function of rate $R$ when traveling the direct distance between the highway and High Tech Place.

h. What constant can be multiplied by the equation in part g to obtain the time in minutes?

i. Let $T_m$ represent the time in minutes. Find a second equation that gives the time in minutes $T_m$ as a function of the rate $R$ to travel the distance between the highway and High Tech Place.
j. Enter the equation from part \( i \) into a graphing calculator, and then use the table feature to complete the following table. This should give you an idea of the time of travel between the highway and High Tech Place at different speeds. Round the time in minutes to the nearest tenth of a minute.

<table>
<thead>
<tr>
<th>Rate (mph)</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

k. Use the data pairs in the table above to graph the time in minutes as a function of the rate in mph. Label each axis and place a title at the top. Verify your work by creating the same graph on your calculator.

l. Use the table and graph to describe how travel time changes as the speed or rate increases from 30 to 90 mph.

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43
**Discovery Activity 15: Alice in Waterland**

Alice just received her first water bill from the town of Waterland. Alice thinks that the expected payment is a bit high, so she asks Mr. Hatter for advice. He observes the following information from her bill.

<table>
<thead>
<tr>
<th>Meter Readings in cubic feet</th>
<th>Consumption in cubic feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous</td>
<td>Current</td>
</tr>
<tr>
<td>196,300</td>
<td>205,500</td>
</tr>
<tr>
<td></td>
<td>9,200</td>
</tr>
</tbody>
</table>

Mr. Hatter tells Alice that she must convert the 9,200 \( f^3 \) to gallons since he only understands volume measurements in gallons. Alice agrees, but she wants Mr. Hatter to be clear that 9,200 \( f^3 \) is just another way to say 9,200 cubic feet and that it is the feet that is cubed, not the 9,200. She looks up the following conversion factor:

1 cubic foot = 7.48 gallons

**a.** Express the conversion factor above as a ratio (in fractional form) of gallons to cubic feet.

**b.** Let the consumption in gallons be represented by the variable \( n \). Write a second ratio that compares the unknown amount of gallons to the known amount of cubic feet.

**c.** Set up and solve a proportion that equates the two ratios found in parts a and b. Make sure that you use the correct units in your set-up.

Mr. Hatter is now comfortable with the measurements in gallons. Unfortunately the table below has the block water usage in cubic feet. The first block, between 0 and 5000 cubic feet of water used, costs $1.80 per 100 cubic feet. The second block between 5000 and 10,000 cubic feet used cost $2.35 per 100 cubic feet, and so on.

<table>
<thead>
<tr>
<th>Block water usage from (cubic feet) to</th>
<th>0–5000</th>
<th>5000–10,000</th>
<th>10,000–20,000</th>
<th>20,000–50,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water rates per 100 cubic feet</td>
<td>$1.80</td>
<td>$2.35</td>
<td>$2.45</td>
<td>$2.55</td>
</tr>
</tbody>
</table>
d. If Alice consumed 9,200 cubic feet of water, then what columns in the table will she need to use to figure out the total payment that must be made?

Let's first solve a proportion based on the first 5000 cubic feet that Alice used. Represent the unknown cost for the first 5000 cubic feet with the variable $x$.

e. Express the price of $1.80 per 100 cubic feet as your first ratio.
   
   First Ratio:

f. Express the cost $x$ for the first 5000 cubic feet used as your second ratio.
   
   Second Ratio:

g. Set up and solve a proportion that equates the first ratio from part e to the second ratio from part f.

h. Now solve a second proportion based on the remaining cubic feet of water above 5000 that Alice used. Represent the unknown cost of the remaining cubic feet with the variable $y$.

i. Use your results from parts g and h to find the total payment Alice must make.